KEY

11.9 - Functions as Power Series

Calculus II

Find a power series representation and the interval of convergence

1. $f(x) = \frac{6}{1+7x^4}$

$\rightarrow f(x) = 6\left[\frac{1}{1+7x^4}\right]$

$= 6\left[\frac{1}{1-(-7x^4)}\right] = 6\sum_{n=0}^{\infty}(-7x^4)^n = \sum_{n=0}^{\infty} 6(-7)^n (x^4)^n$

$= \sum_{n=0}^{\infty} 6(-7)^n x^{4n}$

$\rightarrow |-7x^4| < 1$

$7|x|^4 < 1$

$|x|^4 < 1/7$

$\rightarrow \boxed{-\frac{1}{7^{1/4}} < x < \frac{1}{7^{1/4}}}$

Power : $\boxed{\sum_{n=0}^{\infty} 6(-7)^n x^{4n}}$

2. $f(x) = \frac{x^3}{3-x^2}$

$\rightarrow f(x) = (x^3)\frac{1}{3-x^2} = \frac{x^3}{3}\cdot\frac{1}{1-\frac{1}{3}x^2} = \frac{x^3}{3}\sum_{n=0}^{\infty}\left(\frac{1}{3}x^2\right)^n$

$= \sum_{n=0}^{\infty}\frac{1}{3}x^3\left(\frac{1}{3}\right)^n (x^2)^n$

$\boxed{= \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^{n+1} x^{2n+3}}$

IoC: $\left|\frac{1}{3}x^2\right| < 1$

$|x| < \sqrt{3}$

$\rightarrow \boxed{-\sqrt{3} < x < \sqrt{3}}$

3. $f(x) = \frac{3x^2}{5-2\sqrt[3]{x}}$

$f(x) = 3x^2 \cdot \frac{1}{5-2\sqrt[3]{x}} = \frac{3x^2}{5}\cdot\frac{1}{1-\frac{2}{5}\sqrt[3]{x}}$

$= \frac{3x^2}{5}\sum_{n=0}^{\infty}\left(\frac{2}{5}\sqrt[3]{x}\right)^n$

$\boxed{= \sum_{n=0}^{\infty}\frac{3}{5}\left(\frac{2}{5}\right)^n x^{\frac{1}{3}n+2}}$

IoC: $\left|\frac{2}{5}\sqrt[3]{x}\right| < 1$

$\rightarrow \boxed{\frac{-125}{8} < x < \frac{125}{8}}$

4. $g(x) = \frac{x^3}{1+x^7}$

$$g(x) = x^3 \cdot \frac{1}{1+x^7} = x^3 \cdot \frac{1}{1-(-x^7)} = \sum_{n=0}^{\infty} (-x^7)^n$$

$$= x^3 \sum_{n=0}^{\infty} (-1)^n x^{7n}$$

$$= \sum_{n=0}^{\infty} (-1)^n x^3 x^{7n}$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n x^{7n+3}}$$

IoC: $|-x^7| < 1$

$|x| < 1$

$\boxed{-1 < x < 1}$

5. Use a power series approximate the definite integral, I, to six decimal places.

$\int_0^{0.3} \frac{x^3}{1+x^7} dx$

$$= \int_0^{.3} x^3 \sum_{n=0}^{\infty} (-1)^n x^{7n} \, dx$$

$$= \sum_{n=0}^{\infty} \left[\frac{(-1)^n x^{7n+4}}{7n+4} \right]_0^{.3}$$

$$= \frac{3^4}{4(10^4)} - \frac{3^{11}}{11(10^{11})} + \frac{3^{18}}{18(10^{18})} - \ldots$$

$$\approx .002025$$