Key

Exam 1 Test Prep

1. $\int_1^4 \quad x^3 dx$ $\rightarrow \frac{x^4}{4}\Big|_1^4 = \left[\frac{4^4}{4} - \frac{1^4}{4}\right] = \frac{256}{4} - \frac{1}{4} = \frac{255}{4}$

2. $\int \quad (4x^2 + 2)dx$ $\rightarrow \frac{4x^3}{3} + 2x + C$

3. $\int_0^2 \quad \frac{1}{1+4x^2} dx$ $\rightarrow \frac{1}{a}\arctan(ax) = \left[\frac{1}{2}\arctan(2x)\right]_0^2$

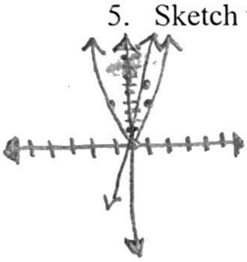
   $\rightarrow a = 2$

   $= \frac{1}{2}(\arctan(4) - \arctan(0))$

   $= \frac{1}{2}\arctan(4)$

4. $\int \quad \cancel{x\cos(x^3)dx}$

5. Sketch the region enclosed by y=$2x^2$ and y=6x. Find the area of the region.



intersection @ $x = 0$, $x = 3$

$A = \int_0^3 (\underset{\text{top}}{6x} - \underset{\text{bottom}}{2x^2})\,dx = \frac{6x^2}{2} - \frac{2x^3}{3}\Big|_0^3 \; 27 - 0 = 27$

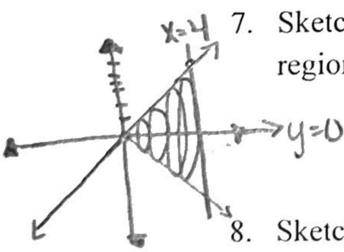
6. Find the x-coordinates of the intersections of the curves $y = 4x\sin(x^2)$ and $y = 4x^2$ for $x \geq 0$. Then find the area of the region between the curves.

$x = 0, \sqrt{\pi}$

$A = \int_0^{\sqrt{\pi}} (4x\sin(x^2) - 4x^2)\,dx = (-2\cos(x^2) - \frac{4x^3}{3}\Big|_0^{\sqrt{\pi}}$

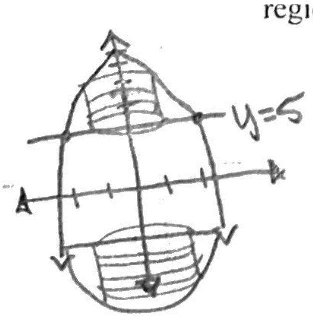
$\rightarrow \left[(-2\cos(\sqrt{\pi})^2 - \frac{4(\sqrt{\pi})^3}{3}) - (-2\cos(0)^2 - \frac{4(0)^3}{3}) = 4 - \frac{4\pi\sqrt{\pi}}{3}\right.$

7. Sketch the region & the solid, then find the volume of the solid formed by revolving the region bounded by $y = x + 2$, $y = 0, x = 0,$ and $x = 4$ about the x-axis.



$V = \pi\int_0^4 (x+2)^2\,dx = \pi\left[\frac{x^3}{3} + 2x^2 + 4x\right]_0^4 = \frac{208}{3}\pi$

8. Sketch the region & the solid, then find the volume of the solid formed by revolving the region bounded by $y = 9 - x^2$ and $y = 5$ about the x-axis.



$V = \pi\int_{-2}^{2} ((9-x^2)^2 - 5^2))\,dx$

$= \pi\int_{-2}^{2} 56 - 18x^2 + x^4\,dx$

$= \pi\left[56x - 6x^3 - \frac{x^5}{5}\right]_{-2}^{2} = \frac{704\pi}{5}$

9. Set up (do not evaluate) the integral for the volume of the solid obtained by revolving the region bounded by $y = cos(x), y = 0, and\ 0 \leq x \leq \frac{\pi}{2}$ about the line y=-4.

Distance

$V = \pi \int_0^{\pi/2} [(\cos(x)+4)^2 - 4^2]dx$

10. Set up (do not evaluate) the integral for the volume of the solid obtained by revolving the region bounded by $y = sin^2(x), y = \frac{1}{4}$, and x=0 about the line y=2.

$\sin^2(x) = \frac{1}{4}$

$\sin(x) \pm 1/2$

$x = \pi/6$

$V = \pi \int_0^{\pi/6} \left(\frac{7}{4}\right)^2 - [2-\sin^2(x)]^2 dx$

$R(x) = 2 - \frac{1}{4} = \frac{7}{4}$

$r(x) = 2 - \sin^2(x)$

11. Find the volume of the solid formed by revolving the region bounded by $y = x^2, y = 0, x = 0$, and $x = 2$ about the y=axis.

$r(x) = 2$

$V = 2\pi \int_0^2 (x)(x^2)dx$

$= 2\pi \int_0^2 x^3 dx$

$= 2\pi \left[\frac{x^4}{4}\right]\Big|_0^2 = 2\pi\left(\frac{16}{4} - \frac{0}{4}\right) = 8\pi$

12. Find the volume of the solid formed by revolving the region bounded by $y = ln(x), y = 0, x = 1$, and $x = e$ about the x-axis. *disk

$V = \pi \int_1^e [\ln(x)]^2 dx = x(\ln^2(x) - 2\ln(x) + 2)$

$= \pi(e-2)$

13. A 1200 lb safe is lifted vertically 25 ft by a crane. Compute the work done.

$W = F \cdot d$

$= 1200 \cdot 25$

$= 30000$ lb-ft

14. Using the midpoint rule with n = 4, estimate the work done from x = 2 to x = 18

x(m)	2	6	10	14	18
f(x)(N)	3.5	5.2	7.8	6.1	4.4

$\Delta X = \frac{18-2}{4} = 4$

Midpt : 2, 6, 10, 14

$(3.5+5.2+7.8+6.1) = 22.6$

$W = Fd \rightarrow 4(22.6) = 90.4\ J$

15. A force of 12 lb is required to hold a spring stretched 6 in beyond its natural length. How much work is done in stretching it from its natural length to 15 in beyond its natural length? Hooke Law $\rightarrow F = kx$

$* \frac{15}{12} = 1.25\ ft$

$* 6in = .5ft$

$12 = k(.5)$

$k = \frac{12}{.5} = 24\ lb/ft$

$W = \int_a^b kx\,dx$

$= \int_0^{1.25} 24x\,dx \rightarrow 12x^2\Big|_0^{1.25} = 18.75\ lbft$

16. Find the average value of the function $h(x) = sin(2x)$ on the interval $[0, \frac{\pi}{2}]$

$\text{Avg Val} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$b-a \rightarrow \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\frac{1}{b-a} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$

$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$

$= \frac{2}{\pi}\left[-\frac{1}{2}\cos(2x)\right]_0^{\pi/2} = \frac{2}{\pi}\cdot 1 = \frac{2}{\pi}$

17. Find the average value of $f(t) = \frac{1}{t^2}$ on the interval [1,3]. Then find c such that $f(c) = f_{avg}$.

$f_{avg} = \frac{1}{3-1}\int_1^3 \frac{1}{t^2}\,dt \rightarrow \frac{1}{2}\int_1^3 t^{-2}\,dt$

$\rightarrow \frac{1}{2}\left[-\frac{1}{t}\right]_1^3 = \frac{1}{2}\cdot\frac{2}{3} = 1/3$

18. A rod 8 m long has a linear density given by $\delta(x) = \frac{x^2}{\sqrt{x+1}}$ kg/m where x is measured from one end. Find the average density of the rod.

$$\delta_{avg} = \boxed{\frac{1}{8}} \int_0^8 \frac{x^2}{\sqrt{x+1}}\,dx$$

$u = x+1$
$x = u-1$
$dx = du$

when $x=0$
$u=1$

when $x=8$
$u=9$

$$= \int_1^9 \frac{(u-1)^2}{\sqrt{u}}\,du = \int_1^9 \frac{u^2-2u+1}{u^{1/2}}\,du$$

$$= \int_1^9 \left(u^{3/2} - 2u^{1/3} + u^{-1/2}\right) du$$

$$= \left[\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + 2u^{1/2}\right]_1^9$$

$$f_{avg} = \frac{1}{8} \cdot \frac{992}{15} = \frac{124}{15} \approx 8.27\,kg/m$$