KEY

Exam 3 Test Prep

Calculus II

1. List the first 5 terms in the sequence, starting at n=1. $\rightarrow a_n = \frac{(-1)^n}{2n-1}$

$a_1 = -1$

$a_2 = 1/3$

$a_3 = -1/5$

$a_4 = 1/7$

$a_5 = -1/9$

2. Find the general form.

$\{\frac{-1}{2}, \frac{4}{3}, \frac{-9}{4}, \frac{16}{5}, ...\}$

$$a_n = \frac{(-1)^n n^2}{n+1}$$

3. Determine if the sequence converges or diverges.

$\lim_{n\to\infty} \frac{2n^2+5n}{n^3+4n^2+1}$

$$\rightarrow \lim_{n\to\infty} \frac{\frac{2n^2}{n^3}+\frac{5n}{n^3}}{\frac{n^3}{n^3}+\frac{4n^2}{n^3}+\frac{1}{n^3}} = \lim_{n\to\infty} \frac{\frac{2}{n}+\frac{5}{n^2}}{1+\frac{4}{n}+\frac{1}{n^3}} = \frac{0+0}{1+0+0} = \frac{0}{1} = 0$$

$\rightarrow$ converge

4. Find the first 3 sums.

$\sum_{n=1}^{\infty} \frac{1}{n^2}$

$s_1 = 1/1^2 = 1$

$s_2 = 1 + 1/2^2 = 1 + \frac{1}{4} = 5/4$

$s_3 = 5/4 + 1/3^2 = 5/4 + 1/9 = 49/36$

5. Use Comparison Test.

$\sum_{n=1}^{\infty} \frac{n}{n^4+n^2+1} \rightarrow \frac{n}{n^4} = \frac{1}{n^3} = b_n$

*we know b_n is a p-series where $p=3>1$

$\rightarrow n^4+n^2+1 > n^4 \rightarrow \frac{1}{n^4+n^2+1} < \frac{n}{n^4} = \frac{1}{n^3}$ $\longrightarrow$ converges

6. Use Ratio Test.

$\sum_{n=1}^{\infty} \frac{n^2 4^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2 4^{n+1}}{(n+1)!}}{\frac{n^2 4^n}{n!}} \right| = \frac{(n+1)^2 4^{n+1}}{(n+1)!} \cdot \frac{n!}{n^2 4^n} = \frac{(n+1)^2 \cancel{4^n} 4}{(n+1)!} \cdot \frac{n!}{n^2 \cancel{4^n}}$$

$$\Rightarrow \frac{4(\cancel{n^2}+2n+1)\cancel{n!}}{((n+1)\cancel{n!}\,\cancel{n^2}} = \frac{4(2n+1)}{n+1} = \frac{4}{n+1} \cdot \frac{2n+1}{n+1} = 0 \cdot 2 = 0$$

$\rightarrow$ converges