KEY

10.1 - Parametric Equations

Calculus II

1. For the given parametric equations, find the points (x,y) corresponding to the parameter values $t = -2, -1, 0, 1, 2$.

$$x = 9t^2 + 9t, y = 3^{t+1}$$

$t = -2 \rightarrow (x, y) =$ $(18, 1/3)$

$t = -1 \rightarrow (x, y) =$ $(0, 1)$

$t = 0 \rightarrow (x, y) =$ $(0, 3)$

$t = 1 \rightarrow (x, y) =$ $(18, 9)$

$t = 2 \rightarrow (x, y) =$ $(54, 27)$

2. **Eliminate the parameter for the given parametric equations and sketch the graph.**

a. $x = 4 - 2t, y = 3 + 6t - 4t^2$

$X = 4 - 2t$

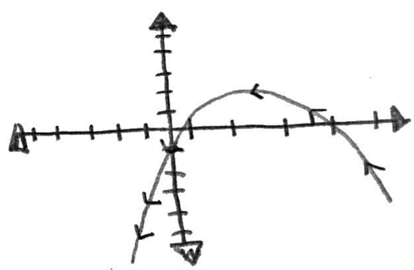
$t = \frac{1}{2}(4 - X)$

$\rightarrow y = 3 + 6[\frac{1}{2}(4-X)] + 4[-\frac{1}{2}(4-X)]^2$

$y = -X^2 + 5X - 1$

$X = \frac{5 \pm \sqrt{21}}{2}$; yint: $(0, -1)$

vertex: $(\frac{5}{2}, \frac{21}{4})$

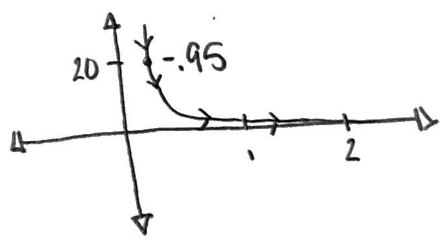


b. $x = \sqrt{t+1}, y = \frac{1}{t+1}, t > -1$

$X = \sqrt{t+1}$

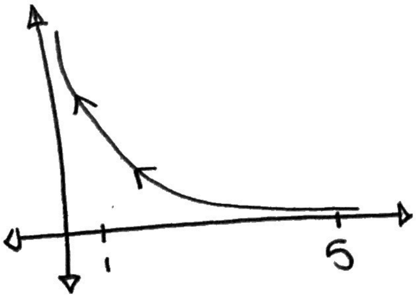
$X^2 = t + 1$

$y = \frac{1}{X^2}$



c. $x = e^{-9t}, y = e^{9t}$

$y = e^{9t} = (e^{-9t})^{-1} = (x)^{-1} = \frac{1}{x}$



10.2 - Parametric Curves

1. Find $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dy}{dx}$

a. $x = 6t^3 + 4t, y = 3t - 5t^2$

$dx/dt = 18t^2 + 4$ $\quad dy/dx = \dfrac{3-10t}{18t^2+4}$

$dy/dt = 3 - 10t$

b. $x = 2te^t, y = 8t + \sin(t)$

$dx/dt = 2te^t + 2e^t$ $\quad dy/dt = 8 + \cos(t)$ $\quad dy/dx = \dfrac{8+\cos(t)}{e^t(2t+2)}$

$= e^t(2t+2)$

2. Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$x = t^5 + 1, y = t^6 + t; t = -1$

$t=-1, (x,y) = (0,0)$

$dy/dt = 6t^5 + 1$ $\quad dy/dx = \dfrac{6t^5+1}{5t^4}$ $\quad dy/dx = \frac{-5}{5} = -1$

$dx/dt = 5t^4$

$y - 0 = -1(x-0)$

$y = -x$

3. Find the surface area generated by rotating the given curve about the y-axis.

$x = 6t^2, y = 4t^3, 0 \le t \le 1$

$u = 1 + t^2$
$du = 2t\,dt$

$(dx/dt)^2 + (dy/dt)^2 = (12t)^2 + (12t^2)^2$

$= 144t^2(1+t^2)$

$\rightarrow \int_0^1 2\pi x \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$

$= \int_0^1 2\pi(6t^2)\sqrt{144t^2(1+t^2)}\,dt = \int_0^1 2\pi(6t^2)12t\sqrt{1+t^2}\,dt$

$= 72\pi\int_1^2 (u-1)\sqrt{u}\,du = 72\pi\int_1^2 (u^{3/2} - u^{1/2})\,du$

$= 24\pi\left(\frac{4\sqrt{2}}{5} + \frac{4}{5}\right)$