KEY

7.8 - Improper Integrals

Calculus II

inf not allowed

1. $\int_0^\infty (1+2x)e^{-x}\,dx$

$\rightarrow \lim_{t\to\infty} \int_0^t (1+2x)e^{-x}\,dx$

$u = 1+2x \rightarrow du = 2dx$
$dv = e^{-x}dx \rightarrow v = -e^{-x}$

$-(1+2x)e^{-x} + 2\int e^{-x}dx$

$-(1+2x)e^{-x} + 2(-e^{-x})$

$-(1+2x)e^{-x} - 2e^{-x} + C$

$(-e^{-x} - 2xe^{-x}) - 2e^{-x} + C$

$-3e^{-x} - 2xe^{-x} + C$

$-e^{-x}(3+2x) + C$

$\rightarrow \lim_{t\to\infty} \left(-e^{-x}(3+2x)\right)\Big|_0^t = \lim_{t\to\infty} \left(3-(3+2t)e^{-t}\right)$

$= \lim_{t\to\infty} 3 - \lim_{t\to\infty} \frac{3+2t}{e^t} = 3 - \lim_{t\to\infty} \frac{2}{e^t} = 3 - 0 = \boxed{3}$

$\rightarrow$ conv.

2. $\int_{-5}^{1} \frac{1}{10+2x}\,dx$

$\rightarrow \lim_{t\to -5^+} \underbrace{\int_t^1 \frac{1}{10+2x}\,dx}$ $= \frac{1}{2}\ln|10+2x| + C$

$\lim_{t\to -5^+} \left(\frac{1}{2}\ln|10+2x|\right)\Big|_t^1 = \lim_{t\to -5^+} \left(\frac{1}{2}\ln|12| - \frac{1}{2}\ln|10+2t|\right)$

$= \frac{1}{2}\ln|2| + \infty = \boxed{\infty}$

diverges!

3. $\int_{-\infty}^{1} \sqrt{6-y}dy$ $= \lim\limits_{t \to -\infty} \int_t^1 \sqrt{6-y}\, dy$

$$= \lim_{t \to -\infty} \left(-\frac{2}{3}(6-y)^{3/2}\right)\Big|_t^1$$

$$= \lim_{t \to -\infty} \left(-\frac{2}{3}(5)^{3/2} + \frac{2}{3}(6+t)^{3/2}\right)$$

$$= -\frac{2}{3}(5)^{3/2} + \infty = \boxed{\infty \text{ diverges}}$$

4. $\int_1^2 \frac{4w}{\sqrt[3]{w^2 04}} dw$ $= \lim\limits_{t \to 2^-} \int_1^t \frac{4w}{\sqrt[3]{w^2-4}}\, dw$

$$= \lim_{t \to 2^-} \left(3(w^2-4)^{2/3}\right)\Big|_1^t$$

$$= \lim_{t \to 2^-} \left(3(t^2-4)^{2/3} - 3(-3)^{2/3}\right)$$

$$= -3(-3)^{2/3}$$

$$= \boxed{-3^{5/3} \text{ converges}}$$