KEY

11.8 - Power Series

Calculus II

Determine the interval and radius of convergence

1. $\sum_{n=1}^{\infty} \frac{6^n}{n}(4x-1)^{n-1}$

$$L = \lim_{n\to\infty}\left|\frac{6^{n+1}(4x-1)^{(n+1)-1}}{n+1}\cdot\frac{n}{6^n(4x-1)^{n-1}}\right| = \lim_{n\to\infty}\left|\frac{6n(4x-1)}{n+1}\right|$$

$$= |4x-1|\lim_{n\to\infty}\frac{6n}{n+1} = 6|4x-1|$$

$$\to 6|4x-1| < 1 \to |x-\tfrac{1}{4}| < \tfrac{1}{24}$$

$$24|x-\tfrac{1}{4}| < 1$$

$\boxed{\text{RoC: } 1/24}$

$$\to -\tfrac{1}{24} < x-\tfrac{1}{4} < \tfrac{1}{24}$$

$$\tfrac{5}{24} < x < \tfrac{7}{24}$$

$x = 5/24: \sum_{n=1}^{\infty}\frac{6^n}{n}\left(-\frac{1}{6}\right)^{n-1} = \sum_{n=1}^{\infty}\frac{6(-1)^{n-1}}{n}$ → alt harmonic ∴ conv

$x = 7/24: \sum_{n=1}^{\infty}\frac{6^n}{n}\left(\frac{1}{6}\right)^{n-1} = \sum_{n=1}^{\infty}\frac{6}{n}$ → pseries ∴ div

$\boxed{\text{IoC: } \frac{5}{24} \le x < \frac{7}{24}}$

2. $\sum_{n=0}^{\infty} \frac{n+1}{(2n+1)!}(x-2)^n$

$$L = \lim_{n\to\infty}\left|\frac{(n+1)+1(x-2)^{n+1}}{(2(n+1)+1)!}\cdot\frac{(2n+1)!}{n+1(x-2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+2)(x-2}{(2n+3)(2n+2)(2n+1)!}\cdot\frac{(2n+1)!}{(n+1}\right| = |x-2|\lim_{n\to\infty}\frac{n+2}{(2n+3)(2n+2)(n+1)} = 0$$

→ since $L=0$; IoC: $(-\infty,\infty)$

RoC: ∞

3. $\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

$$L = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)^4 4^{(n+1)}}\cdot\frac{n^4 4^n}{x^n}\right| = \lim_{n\to\infty}\left|\frac{x}{4}\cdot\left(\frac{n}{n+1}\right)^4\right| = \frac{|x|}{4}\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^4$$

$$= \frac{|x|}{4}\cdot 1^4 = \frac{|x|}{4}$$

$$\to \frac{|x|}{4} < 1$$

$$|x| < 4$$

$\boxed{\text{RoC: } 4}$

$\boxed{\text{IoC: } [-4, 4]}$

$x=-4 \to \sum_{n=1}^{\infty}\frac{(-4)^n}{n^4 4^n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}$

conv.

$x=4 \to \sum_{n=1}^{\infty}\frac{4^n}{n^4 4^n} = \sum_{n=1}^{\infty}\frac{1}{n^4}$

conv.

4. $\sum_{n=0}^{\infty} \frac{(x-8)^n}{n^4+1}$

divide everything by n^4

$$L = \lim_{n\to\infty} \left| \frac{(x-8)^{n+1}}{(n+1)^4+1} \cdot \frac{n^4+1}{(x-8)^n} \right| = \lim_{n\to\infty} \left| (x-8) \frac{n^4+1}{(n+1)^4+1} \right| = |x-8| \lim_{n\to\infty} \frac{n^4+1}{n^4+4n^3+6n^2+4n+2}$$

$= |x-8| < 1$

$-1 < x-8 < 1$

$7 < x < 9$

$x=9: \frac{(9-8)^n}{n^4+1} = \frac{1^n}{n^4+1}$

$\to$ conv (direct comp)

$x=7: \sum_{n=0}^{\infty} \frac{(7-8)^n}{n^4+1} = \frac{-1^n}{n^4+1}$

$\to$ conv

$\boxed{\text{RoC}: 1 \quad \text{IoC}: [7,9]}$

5. $\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln(n)}$

$$L = \lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{2^{n+1} \ln(n+1)} \cdot \frac{2^n \ln(n)}{(x+2)^n} \right| = \lim_{n\to\infty} \left| \frac{x+2}{2} \cdot \frac{\ln(n)}{\ln(n+1)} \right| \text{* L'Hopital}$$

$\lim_{n\to\infty} \frac{\ln(n)}{\ln(n+1)} = 1 \; ; \; L = \left|\frac{x+2}{2}\right| \cdot 1 = \frac{|x+2|}{2}$

$\to \frac{|x+2|}{2} < 1 \; ; \; |x+2| < 2 : \; -2 < x+2 < 2$

$-4 < x < 0$

$x=0: \sum_{n=2}^{\infty} \frac{(0+2)^n}{2^n \ln(n)} = \frac{2}{2^n \ln(n)} = \frac{1}{\ln(n)}$

$\to$ div (harmonic)

$x=-4: \sum_{n=2}^{\infty} \frac{(-4+2)^n}{2^n \ln(n)} = \frac{-2^n}{2^n \ln(n)} = \frac{(-1)^n}{\ln(n)}$

$\to$ conv (alt series)

$\boxed{\text{RoC}: 2 \quad \text{IoC}: [-4,0)}$

6. $\sum_{n=1}^{\infty} \frac{x^n}{n4^n}$

$$L = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)4^{n+1}} \cdot \frac{n4^n}{x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x}{4} \cdot \frac{n}{n+1} \right| = \frac{|x|}{4} \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = \frac{|x|}{4}$$

$\frac{|x|}{4} < 1$

$|x| < 4$

$x=4: \sum_{n=1}^{\infty} \frac{4^n}{n4^n} = \frac{1}{n}$

$\therefore$ diverge

$x=-4: \sum_{n=1}^{\infty} \frac{(-4)^n}{n4^n} = \frac{(-1)^n}{n}$

$\therefore$ conv

$\boxed{\text{RoC}: 4 \quad \text{IoC}: [-4,4)}$