KEY!

11.1 - Sequences

Calculus II

1. List the first 5 terms of each sequence.

a. $\frac{4n}{n^2-7}$ starting at n=0

$a_0 = \frac{4(0)}{0^2-7} = \frac{0}{-7} = 0$

$a_1 = \frac{4(1)}{1^2-7} = -\frac{2}{3}$

$a_2 = \frac{4(2)}{2^2-7} = \frac{8}{-3}$

$a_3 = \frac{4(3)}{3^2-7} = 6$

$a_4 = \frac{4(4)}{4^2-7} = \frac{16}{9}$

b. $\frac{(-1)^{n+1}}{2n+(-3)^n}$ starting at n=2

$a_2 = \frac{(-1)^{2+1}}{2(2)+(-3)^2} = \frac{(-1)^3}{4+9} = \frac{-1}{13}$

$a_3 = \frac{(-1)^{3+1}}{2(3)+(-3)^3} = \frac{(-1)^4}{6+(-27)} = \frac{1}{-21}$

$a_4 = -1/89$

$a_5 = -1/233$

$a_6 = -1/741$

2. Determine the rule for the sequence.

a. {4, -12, 36, -108, 324...}

$a_1 = 4$

$r = \frac{-12}{4} = -3$ ← ratio

$a_n = a_1(r)^{n-1}$

$= 4(-3)^{n-1}$

b. {2,11,20,29,38,..}

+9

$a_1 = 2$

com. difference: $11-2 = 9$

$a_n = a_1 + d(n-1)$

$= 2 + 9(n-1)$

$= 2 + 9n - 9 \rightarrow 9n-7$

$\{n+9\}_{n=1}^{\infty}$

c. {15,12,9,6,3,...}

$a_1 = 15$

$d = 12-15 = -3$

$a_n = a_1 + d(n-1)$

$= 15 - 3(n-1) \rightarrow 15 + (-3)(n-1)$

$= 15 - 3n + 3$

$= -3n + 18$

3. Determine whether the sequence converges or diverges. If it converges, find the limit.

a. $\lim_{n\to\infty} \frac{3n^2-1}{5n^2+4n}$ $= \lim_{n\to\infty} \dfrac{\frac{3n^2}{n^2} - \frac{1}{n^2}}{\frac{5n^2}{n^2} + \frac{4n}{n^2}} = \dfrac{3-\frac{1}{n^2}}{5+\frac{4}{n}} = \dfrac{3-0}{5+0}$

$= 3/5$

→ converges

b. $\lim_{n\to\infty} (-1)^{n+1} \frac{2n}{n^2+1}$ $= \lim_{n\to\infty} \dfrac{\frac{2n}{n^2}}{\frac{n^2}{n^2} + \frac{1}{n^2}}$

→ $\lim_{n\to\infty} \dfrac{\frac{2}{n}}{1+\frac{1}{n^2}} = \dfrac{0}{1+0} = \dfrac{0}{1} = 0$

→ converges

c. $\lim_{n\to\infty} \frac{n^2-7n+3}{1+10n-4n^2}$

$\lim_{n\to\infty} \dfrac{\frac{n^2}{n^2} - \frac{7n}{n^2} + \frac{3}{n^2}}{\frac{1}{n^2} + \frac{10n}{n^2} - \frac{4n^2}{n^2}} = \dfrac{1-\frac{7}{n}+\frac{3}{n^2}}{\frac{1}{n^2}+\frac{10}{n}-4} = \dfrac{1-0+0}{0+0-4}$

$= -1/4$

→ conv

d. $\cos(n\pi)$

$a_1 = \cos(1\pi) = -1$

$a_2 = \cos(2\pi) = 1$

$a_3 = \cos(3\pi) = -1$

$a_4 = \cos(4\pi) = 1$

→ diverges