KEY

8.1 - Arc Length

Calculus II

1. Set up but do not evaluate, the integral for the length of the curve.

$x = 9sin(y), 0 \leq y \leq \frac{\pi}{2}$

$$x = 9\sin(y) \to \frac{dx}{dy} = 9\cos(y) \to 1 + \left(\frac{dx}{dy}\right)^2 = 1 + 81\cos^2(y)$$

$$L = \int_0^{\pi/2} \sqrt{1+81\cos^2(y)}\, dy$$

2. Find the exact length of the curve

$y = \frac{2}{3}x^{\frac{3}{2}}, 0 \leq x \leq 4$

$$y = \frac{2}{3}x^{3/2} \to \frac{dy}{dx} = x^{1/2} \to 1 + (dy/dx)^2 = 1 + x$$

$$L = \int_0^4 \sqrt{1+x}\, dx = \int_0^4 (1+x)^{1/2} dx$$

$$= \int_1^5 u^{1/2} du = \left[\frac{u^{3/2}}{3/2}\right]_1^5 = \frac{2}{3}\left[u^{3/2}\right]_1^5$$

$$= \frac{2}{3}(5\sqrt{5} - 1)$$

$u = 1+x$
$du = dx$
$x = 0 \to u = 1$
$x = 4 \to u = 5$

3. Find the exact length of the curve.

$x = e^y + \frac{1}{4}e^{-y}, 0 \leq y \leq 7$

$$x = e^y + \frac{1}{4}e^{-y} \to dy/dx = e^y - \frac{1}{4}e^{-y} \to 1 + (dy/dx)^2 = \left(e^y + \frac{1}{4}e^{-y}\right)^2$$

$$L = \int_0^7 \sqrt{\left(e^y + \frac{1}{4}e^{-y}\right)^2}\, dy = \int_0^7 e^y + \frac{1}{4}e^{-y}\, dy$$

$$= \left[e^y - \frac{1}{4}e^{-y}\right]_0^7$$

$$= e^7 - \frac{1}{4e^7} - \frac{3}{4}$$

4. Find the exact length of the curve.

$y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x), 1 \leq x \leq 2$

$y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x) \to dy/dx = \frac{1}{2}x - \frac{1}{2x} \to 1 + (dy/dx)^2$

$= \frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}$

$= \left(\frac{1}{2}x + \frac{1}{2x}\right)^2$

$L = \int_1^2 \sqrt{\left(\frac{1}{2}x + \frac{1}{2x}\right)^2}\, dx$

$= \int_1^2 \sqrt{(1/2x + 1/2x)}\, dx = \left[x^2/4 + 1/2\ln(x)\right]_1^2$

$= \frac{3}{4} + \frac{1}{2}\ln(x)$

This is a hard problem

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5. Determine the length of:

$y = 7(6 + x)^{3/2}, 189 \leq y \leq 875$

$y = 7(6+x)^{3/2} \to dy/dx = \frac{21}{2}(6+x)^{1/2}$

Lower lim: $189 = 7(6+x)^{3/2}$

$27 = (6+x)^{3/2}$

$x_1 = 3$

$1 + (dy/dx)^2 = \frac{441}{4}(x+6) + 1$

$= \frac{441}{4}x + \frac{1325}{2}$

Upper lim: $875 = 7(6+x)^{3/2}$

$125 = (6+x)^{3/2}$

$x_2 = 19$

$L = \int_3^{19} \sqrt{\frac{441x}{4} + \frac{1325}{2}}\, dx = \frac{1}{2}\int_3^{9} \sqrt{441x + 2650}\, dx$

$= \frac{1}{1323}\left[2650 + 441x\right]_3^9$

$= \frac{1}{1323}\left(11029^{3/2} - 3973^{3/2}\right)$

≈ 686.19