

7.4 - Partial Fractions

Calculus II - KEY

$$1. \int \frac{5x-1}{x^2-2x-15} dx \rightarrow \int \frac{5x-1}{(x-5)(x+3)} dx = \frac{A}{x-5} + \frac{B}{x+3}$$

$$\rightarrow 5x-1 = A(x+3) + B(x-5)$$

$$5x-1 = Ax+3A+Bx-5B$$

$$5x-1 = x(A+B) + (3A-5B)$$

$$A+B=5$$

$$3A-5B=-1$$

$$\frac{5}{3}B - \frac{1}{3} + B = 5$$

$$\frac{3A}{3} = \frac{5B-1}{3}$$

$$\frac{5}{3}B + B = \frac{16}{3}$$

$$A = \frac{5}{3}B - \frac{1}{3}$$

$$\frac{8}{3}B = \frac{16}{3}$$

$$B = 2$$

$$A = \frac{5}{3}(-2) - \frac{1}{3}$$

$$A = -3$$

$$\rightarrow \int \frac{-3}{x-5} + \frac{2}{x+3} dx = -3 \ln|x-5| + 2 \ln|x+3| + C$$

$$= -3 \ln|x-5| + 2 \ln|x+3| + C$$

$$2. \int \frac{-x^2+2x+4}{x^3-4x^2+4x} = \int \frac{-x^2+2x+4}{x(x^2-4x+4)} = \int \frac{-x^2+2x+4}{x(x-2)^2}$$

$$= \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \rightarrow -x^2+2x+4 = A(x-2)^2 + B(x)(x-2) + C(x)$$

$$\rightarrow -x^2+2x+4 = A(x^2-4x+4) + B(x^2-2x) + C(x)$$

$$-x^2+2x+4 = Ax^2-4xA+4A+Bx^2-2xB+Cx$$

$$-x^2+2x+4 = x^2(A+B) + x(-4A-2B+C) + 4A$$

$$\rightarrow A+B=-1$$

$$-4A-2B+C=2$$

$$4A=4$$

$$1+B=-1$$

$$-4(1)-2(-2)+C=2$$

$$A=1$$

$$B=-2$$

$$-4+4+C=2$$

$$C=2$$

$$A=1; B=-2; C=2$$

$$= \int \frac{1}{x} - \frac{2}{x-2} + \frac{2}{(x-2)^2} dx$$

$$= \ln|x| - 2\ln|x-2| - \frac{2}{x-2} + C$$

$$3. \int \frac{2+w^4}{w^3+9w} dw = \int \frac{2-9w^2}{w(w^2+9)} dw$$

$$\rightarrow \frac{2-9w^2}{w^2+9} = \frac{A}{w} + \frac{Bw+C}{w^2+9}$$

$$2-9w^2 = A(w^2+9) + (Bw+C)(w)$$

$$2-9w^2 = Aw^2 + 9A + Bw^2 + Cw$$

$$2-9w^2 = w^2(A+B) + w(C) + 9A$$

$$\rightarrow A+B=-9 \quad C=0 \quad 9A=2$$

$$\rightarrow 2/9 + B = -9 \quad A = 2/9$$

$$B = -83/9$$

$$\rightarrow \int \frac{\frac{2}{9}}{w} + \frac{-83/9 + 0}{w^2+9} dw = \frac{w^2}{2} + \frac{2}{9} \ln|w| - \frac{83}{18} \ln|w^2+9| + C$$

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$$4. \int \frac{z^2 + 2z + 3}{(z-6)(z^2+4)} dz = \frac{A}{z-6} + \frac{Bz+C}{z^2+4}$$

$$\begin{aligned} \rightarrow z^2 + 2z + 3 &= A(z^2+4) + (Bz+C)(z-6) \\ &= Az^2 + 4A + Bz^2 - 6Bz + Cz - 6C \\ &= z^2(A+B) + z(-6B+C) + (4A-6C) \end{aligned}$$

$$A+B=1$$

$$-6B+C=2$$

$$4A-6C=3$$

$$\frac{3}{2}C + \frac{3}{4} + \frac{1}{3} + \frac{C}{6} = 1 \quad -6B = 2-C$$

$$4A = 6C + 3$$

$$A = \frac{3}{2}C + \frac{3}{4}$$

$$\frac{5}{3}C + \frac{5}{12} = 1$$

$$B = -\frac{1}{3} + \frac{C}{6}$$

$$A = \frac{3}{2}\left(\frac{7}{20}\right) + \frac{3}{4}$$

$$\frac{5}{3}C = \frac{7}{12}$$

$$B = -\frac{1}{3} + \frac{7}{20 \cdot 6}$$

$$A = 51/40$$

$$C = 7/20$$

$$B = -11/40$$

$$\int \frac{51}{40} \frac{1}{z-6} + \frac{-11}{40} \frac{z}{z^2+4} + \frac{7}{20} \frac{1}{z^2+4} dz \rightarrow \int \frac{51}{40} \frac{1}{z-6} - \frac{11}{40} \frac{z}{z^2+4} + \frac{7}{20} \frac{1}{z^2+4} dz$$

$$= \frac{51}{40} \ln|z-6| - \frac{11}{80} \ln|z^2+4| + \frac{7}{40} \tan^{-1}\left(\frac{z}{2}\right) + C$$

$$5. \int \frac{x^2 - 29x + 5}{(x-4)^2(x^2+3)} dx = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+3}$$

$$\rightarrow x^2 - 29x + 5 = A(x-4)(x^2+3) + B(x^2+3) + (Cx+D)(x-4)^2$$

$$= Ax^3 - 4x^2A + 3xA - 12A + Bx^2 + 3B + Cx^3 - 8Cx^2 + Dx^2 + 16Cx - 8Dx + 16D$$

$$x^2 - 29x + 5 = x^3(A+C) + x^2(-4A+B-8C+D) + x(3A+16C-8D) + (-12A+3B+16D)$$

$$A+C=0$$

$$-4A+B-8C+D=1$$

$$3A+16C-8D=-29$$

$$A=-C$$

$$-4A+B-8(-A)+D=1$$

$$3A+16(-A)-8D=-29$$

$$C=-A$$

$$-4A+8A+B+D=1$$

$$3A-16A-8D=-29$$

$$-12A+3B+16D=5$$

$$4A+B+D=1$$

$$-13A-8D=-29$$

$$-12A+3(1-4A-D)+16D=5$$

$$B=1-4A-D$$

$$-12A+3-12A-3D+16D=5$$

$$-24A+3+13D=5$$

$$-13A-8D=-29$$

$$-24A+13D=2$$

$$\rightarrow \int \frac{1}{x-4} + \frac{-5}{(x-4)^2} + \frac{-x+2}{x^2+3} dx$$

$$= \ln|x-4| + \frac{5}{x-4} - \frac{1}{2} \ln(x^2+3) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

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$$6. \int \frac{x^4 - 5x^3 + 6x^2 - 18}{x^3 - 3x^2} dx = \int \frac{x^4 - 5x^3 + 6x^2 - 18}{x^2(x-3)} dx$$

$$x^2(x-3) \overline{) x^4 - 5x^3 + 6x^2 - 18}$$

$$\hookrightarrow x - 2 + \frac{-18}{x^2(x-3)}$$

$$\frac{-18}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

$$-18 = A(x)(x-3) + B(x-3) + C(x^2)$$

$$-18 = A(x^2 - 3x) + B(x-3) + C(x^2)$$

$$-18 = Ax^2 - 3xA + Bx - 3B + Cx^2$$

$$-18 = x^2(A+C) + x(-3A+B) - 3B$$

$$A+C=0 \quad -3A+B=0 \quad -3B=-18$$

$$2+C=0 \quad -3A+6=0 \quad B=6$$

$$C=-2 \quad -3A=-6$$

$$A=2$$

$$\int \frac{2}{x} + \frac{6}{x^2} - \frac{2}{x-3} + (x-2) dx$$

$$= 2\ln|x| - \frac{6}{2} - 2\ln|x-3| + \frac{x^2}{2} - 2x$$