KEY

11.2 - Series

Calculus II

Find the first 3 terms.

1. $\sum_{n=1}^{\infty} n2^n$

$S_1 = (1)2^1 = 2$

$S_2 = (1)2^1 + (2)2^2 = 2+8=10$

$S_3 = (1)2^1 + (2)2^2 + (3)2^3 = 34$

2. $\sum_{n=3}^{\infty} \frac{2n}{n+2}$

$S_3 = \frac{2(3)}{3+2} = 6/5$

$S_4 = \frac{2(3)}{3+2} + \frac{2(4)}{4+2} = 38/15$

$S_5 = \frac{2(3)}{3+2} + \frac{2(4)}{4+2} + \frac{2(5)}{5+2} = \frac{416}{105}$

Determine if the series is convergent or divergent. If convergent, solve.

3. $s_n = \frac{5+8n^2}{2-7n^2}$

$= \lim_{n\to\infty} \frac{5+8n^2}{2-7n^2} = -\frac{8}{7}$

$\to$ converges

4. $s_n = \frac{n^2}{5+2n}$

$= \lim_{n\to\infty} \frac{n^2}{5+2n} = \lim_{n\to\infty} \frac{\frac{n^2}{n}}{\frac{5}{n}+\frac{2n}{2}} = \frac{n}{\frac{5}{n}+2} = \frac{\infty}{0+2} = \frac{\infty}{2} = \infty$

$\to$ diverges

Show that the series is divergent.

5. $\sum_{n=0}^{\infty} \frac{3ne^n}{n^2+1}$

$\lim_{n\to\infty} \frac{3ne^n}{n^2+1}$ * as $n\to\infty$ $3ne^n \to \infty$ $n^2+1 \to \infty$ $\lim_{n\to\infty} \frac{3ne^n}{n^2+1} = \frac{\infty}{\infty}$

$\infty \neq 0 \to$ diverges

6. $\sum_{n=5}^{\infty} \frac{6+8n+9n^2}{3+2n+n^2}$

$\lim_{n\to\infty} \frac{6+8n+9n^2}{3+2n+n^2} = \frac{9}{1} = 9$

$\to 9 \neq 0 \to$ diverges

Geo: $\sum_{n=0}^{\infty} ar^n \quad S=\frac{a}{1-r}$

Special Series - determine if it converges or diverges.

7. $\sum_{n=0}^{\infty} 3^{2+n}2^{1-3n}$ - Geometric

$=\sum_{n=0}^{\infty} 3^2 3^n 2^1 2^{-3n}$

$=9(2)\frac{3^n}{2^{3n}} = \sum_{n=0}^{\infty} 18\frac{3^n}{(2^3)^n} = \sum_{n=0}^{\infty} 18\cdot\frac{3^n}{8^n} = 18\left(\frac{3}{8}\right)^n$

$a=18$
$r=3/8$
$|r|=\frac{3}{8}<1$
→ converges

$=\frac{18}{1-3/8} = \boxed{\frac{144}{5}}$

8. $\sum_{n=1}^{\infty} \frac{5}{6n}$

Harmonic
→ diverges

9. $\sum_{n=1}^{\infty} \frac{3}{n^2+7n+12}$ - Telescoping

$\frac{3}{(n+3)(n+4)} = \frac{A}{n+3}+\frac{B}{n+4} \rightarrow 3=A(n+4)+B(n+3)$

$3=An+4A+Bn+3B$

$3=n(A+B)+(4A+3B)$

$A+B=0 \qquad 4A+3B=3$

$A=3, B=-3$

$\sum_{n=1}^{\infty}\left(\frac{3}{n+3}-\frac{3}{n+4}\right)$

$=\left(\frac{3}{4}-\frac{3}{5}\right)+\left(\frac{3}{5}-\frac{3}{6}\right)+\left(\frac{3}{6}-\frac{3}{7}\right)\ldots$

$=\boxed{3/4}$ → converges

10. $\sum_{n=1}^{\infty} \frac{5^{n+1}}{7^{n-2}}$ - geo

$\sum_{n=1}^{\infty}\frac{5^{n-1}5^2}{7^{n-1}7^{-1}} = (25)(7)\frac{5^{n-1}}{7^{n-1}} = \sum_{n=1}^{\infty} 175\left(\frac{5}{7}\right)^{n-1}$

$a=175$
$r=5/7$
$|r|=\frac{5}{7}<1$

$\rightarrow \frac{175}{1-\frac{5}{7}} = \boxed{\frac{1225}{2}}$

→ conv.

11. $\sum_{n=4}^{\infty} \frac{10}{n^2-4n+3}$ - telescoping

$\sum_{n=4}^{\infty}\frac{10}{(n-1)(n-3)} = \frac{A}{n-1}+\frac{B}{n-3} \rightarrow 10=A(n-3)+B(n-1)$

$10=An-3A+Bn-B$

$10=n(A+B)+(-3A-B)$

$A+B=0 \qquad -3A-B=10$

$A=-5, B=5$

$\sum_{n=4}^{\infty}\left(\frac{-5}{n-1}+\frac{5}{n-3}\right)$

$\rightarrow\left(\frac{-5}{3}+\frac{5}{1}\right)+\frac{-5}{}+\frac{5}{2}\Big)\ldots$

$=15/2$