7.2 Trig Integrals
KEY

1. $\int \sin^8(3z)\cos^5(3z)dz$

$= \int \sin^8(3z)\cos^4(3z)\cos(3z)dz$

Now we use the trig identity $\sin^2\theta + \cos^2\theta = 1$ **to convert the remaining cos to sin.**

$= \int \sin^8(3z)[\cos^2(3z)]^2\cos(3z)dz$

$= \int \sin^8(3z)[1-\sin^2(3z)]^2\cos(3z)dz$

Let $u = \sin(3z)$

$= \frac{1}{3}\int u^8[1-u^2]^2du$

$= \frac{1}{3}\int u^8 - 2u^{10} + u^{12}du$

$= \frac{1}{3}(\frac{1}{9}u^9 - \frac{2}{11}u^{11} + \frac{1}{13}u^{13} + C$

2. $\int \cos^4(2t)dt$

Since all the exponents are even, we use a half-angle

$= \int (\cos^2(2t))^2dt$

$= \int [\frac{1}{2}(1+\cos(4t))^2dt$

$= \int \frac{1}{4}(1+2\cos(4t)+\cos^2(4t))dt$

Use half angle formula again

$= \frac{1}{4}\int 1+2\cos(4t)+\frac{1}{2}[1+\cos(8t)]dt$

$= \frac{1}{4}\int \frac{3}{2}+2\cos(4t)+\frac{1}{2}\cos(8t)dt$

$= \frac{1}{4}(\frac{3}{2}t+\frac{1}{2}\sin(4t)+\frac{1}{16}\sin(8t))+C$

$= \frac{3}{8}t+\frac{1}{8}\sin(4t)+\frac{1}{64}\sin(8t)+C$

3. $\int \frac{2+7\sin^3(z)}{\cos^2(z)}dz$

To make it easier, split fraction up as so

$= \int \frac{2}{\cos^2(z)} + \frac{7\sin^3}{\cos^2(z)}dz$

$= \int \frac{2}{\cos^2(z)} + \int \frac{7\sin^3(z)}{\cos^2(z)}dz$

Rewrite the first integral

$= \int 2\sec^2(z)dz + 7\int \frac{\sin^2(z)}{\cos^2(z)}\sin(z)dz$

Use trig identity for the second integral

$= \int 2\sec^2(z)dz + 7\int \frac{1-\cos^2(z)}{\cos^2(z)}\sin(z)dz$

Use substitution, let $u = \cos(z)$

$= 2\tan(z) - 7\int \frac{1-u^2}{u^2}du$

$= 2\tan(z) - 7\int u^{-2} - 1du$

$= 2\tan(z) - 7(-u^{-1} - u) + C$

4. $\int_{\pi}^{2\pi} \cos^3(\frac{1}{2}w)\sin^5(\frac{1}{2}w)dw$

Make the $\cos^3$ **even**

$= \int \cos^2(\frac{1}{2}w)\sin^5(\frac{1}{2}w)\cos(\frac{1}{2}w)dw$

$= \int (1-\sin^2(\frac{1}{2}w))\sin^5(\frac{1}{2}w)\cos(\frac{1}{2}w)dw$

Let $u = \sin(\frac{1}{2}w)$

$= 2\int (1-u^2)u^5du$

$= 2\int u^5 - u^7du$

$= 2(\frac{u^6}{6} - \frac{u^8}{8}) + C$

Sub u back in!

$= 2(\frac{(\sin(\frac{1}{2}w))^6}{6} - \frac{(\sin(\frac{1}{2}w))^8}{8}) + C$

$= \frac{1}{3}\sin^6(\frac{1}{2}w) - \frac{1}{4}\sin^8(\frac{1}{2}w) + C$ **Now evaluate from** π **tp** 2π

$= -\frac{1}{12}$

5. $\int_1^3 \sin(8x)\sin(x)dx$

$\int_1^3 \frac{1}{2}[\cos(8x-x) - \cos(8x+x)]dx$

$= \frac{1}{2}\int_1^3 \cos(7x) - \cos(9x)dx$

Now evaluate the integral

$= \frac{1}{2}[\frac{1}{7}\sin(7x) - \frac{1}{9}\sin(9x)$

Now evaluate from 1 to 3

$= \frac{1}{14}\sin(21) - \frac{1}{18}\sin(27) - \frac{1}{14}\sin(7) + \frac{1}{18}\sin(9) = -0.0174$

6. $\int_0^{\frac{\pi}{4}} \tan^7(z)\sec^3(z)dz$

Notice we have an odd exponent on the $\tan$ **and can you the following trig identity:**

$\tan^2\theta + 1 = \sec^2\theta$

$= \int \tan^6(z)\sec^2(z)\tan(z)\sec(z)dz$

$= \int [\tan^2(z)]^3\sec^2(z)\tan(z)\sec(z)dz$

$= \int [\sec^2(z) - 1]^3 \sec^2(z) \tan(z) \sec(z) dz$

Now we can use substitution; let $u = \sec(z)$

$= \int [u^2 - 1]^3 u^2 du$

$= \int u^8 - 3u^6 + 3u^4 - u^2 du$

$= \frac{u^9}{9} - \frac{3u^5}{7} + \frac{3u^5}{5} - \frac{u^3}{3} + C$

Sub u back in!

$= \frac{\sec(z)^9}{9} - \frac{3(\sec(z))^5}{7} + \frac{3(\sec(z))^5}{5} - \frac{\sec(z)^3}{3} + C$

Now evaluate from 0 to $\frac{\pi}{4}$

$= \frac{2}{315}(8 + 13\sqrt{2}) = 0.1675$