KEY

7.7 - Approximate Integration

Calculus II

7.5 - Strategy of Integration

Name what strategy you would use for the following problems:

1. $\int xe^x dx$

→ Integration by parts

2. $\int \cos^4(2t)dt$

→ Trig Integrals

3. $\int \sqrt{1-7w^2}dw$

→ Trig Sub

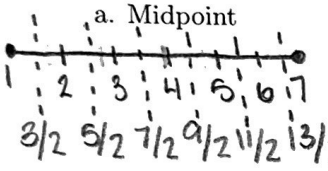
4. $\int \frac{x^4-5x^3+6x^2-18}{x^3-3x^2}dx$

→ Partial Fractions

7.7 - Use the Midpoint, Trapezoid, and Simpson's Rule for the following integrals

1. $\int_1^7 \frac{1}{x^3+1}dx; n=6$ $\Delta X = \frac{7-1}{6} = \frac{6}{6} = 1$

a. Midpoint



$\approx 1 \cdot \left[f\left(\frac{3}{2}\right)+f\left(\frac{5}{2}\right)+f\left(\frac{7}{2}\right)+f\left(\frac{9}{2}\right)+f\left(\frac{11}{2}\right)+f\left(\frac{13}{2}\right)\right]$

$\approx .33197$

b. Trapezoid

$T_n = \Delta X \frac{1}{2}[f(x_0)+2f(x_2)+2f(x_3)\ldots]$

$T_6 = 1\left(\frac{1}{2}\right)[f(1)+2f(2)+2f(3)+2f(4)+2f(5)+2f(6)+f(7)]$

$\approx .426$

c. Simpson's

$S_n = \Delta X\left(\frac{1}{3}\right)[f(x_0)+4f(x_1)+2f(x_2)\ldots]$

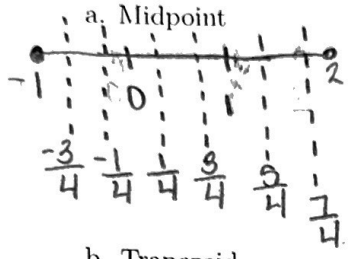
$S_6 = 1\left(\frac{1}{3}\right)[f(1)+4f(2)+2f(3)+4f(4)+2f(5)+4f(6)+f(7)]$

$\approx .3715$

$$\Delta x = \frac{2-(-1)}{6} = \frac{3}{6} = \frac{1}{2}$$

2. $\int_{-1}^{2} \sqrt{e^{-x^2}+1}\,dx;\ n=6$

a. Midpoint



$$M_6 = \frac{1}{2}\left[f(-3/4)+f(-1/4)+f(1/4)+f(3/4)+f(5/4)+f(7/4)\right]$$

$$\approx 3.7070$$

b. Trapezoid

$$T_6 = \frac{1}{2}\left(\frac{1}{2}\right)\left[f(-1)+2f(-\tfrac{1}{2})+2f(0)+2f(\tfrac{1}{2})+2f(1)+2f(\tfrac{3}{2})+f(2)\right]$$

$$\approx 3.6946$$

c. Simpson's

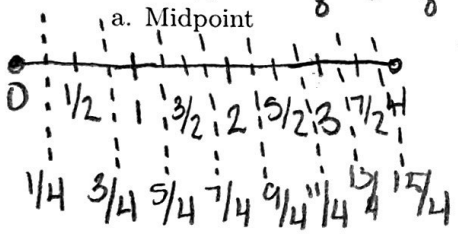
$$S_6 = \frac{1}{3}\left(\frac{1}{2}\right)\left[f(-1)+4f(-\tfrac{1}{2})+2f(0)+4f(\tfrac{1}{2})+2f(1)+4f(\tfrac{3}{2})+f(2)\right]$$

$$\approx 3.7035$$

3. $\int_{0}^{4} \cos(1+\sqrt{x})\,dx;\ n=8$

$$\Delta x = \frac{4-0}{8} = \frac{4}{8} = \frac{1}{2}$$

a. Midpoint



$$M_8 = \frac{1}{2}\left[f(\tfrac{1}{4})+f(\tfrac{3}{4})+f(\tfrac{5}{4})+f(\tfrac{7}{4})+f(\tfrac{9}{4})+f(\tfrac{11}{4})+f(\tfrac{13}{4})+f(\tfrac{15}{4})\right]$$

$$\approx -2.516$$

b. Trapezoid

$$T_8 = \frac{1}{2}\left(\frac{1}{2}\right)\left[f(0)+2f(\tfrac{1}{2})+2f(1)+2f(\tfrac{3}{2})+2f(2)+2f(\tfrac{5}{2})+2f(3)+2f(\tfrac{7}{2})+f(4)\right]$$

$$\approx -2.4300$$

c. Simpson's

$$S_6 = \frac{1}{2}\left(\frac{1}{3}\right)\left[f(0)+4f(\tfrac{1}{2})+2f(1)+4f(\tfrac{3}{2})+2f(2)+4f(\tfrac{5}{2})+2f(3)+4f(\tfrac{7}{2})+f(4)\right]$$

$$\approx -2.4716$$