KEY

11.6 - Ratio and Root Test

Calculus II

$3^{1-2n-2} \to 3^{-1-2n}$

Ratio Test - converges or diverges?

1. $\sum_{n=1}^{\infty} \frac{3^{1-2n}}{n^2+1}$ $\to L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{3^{1-2(n+1)}}{(n+1)^2+1}}{\frac{3^{1-2n}}{n^2+1}}\right| = \lim_{n\to\infty} \left|\frac{3^{1-2(n+1)}}{(n+1)^2+1} \cdot \frac{n^2+1}{3^{1-2n}}\right|$

$$= \lim_{n\to\infty} \left|\frac{1}{(n+1)^2+1} \cdot \frac{n^2+1}{3^2}\right|$$

$$= \lim_{n\to\infty} \left|\frac{n^2+1}{9[(n+1)^2+1]}\right| = 1/9 \therefore \text{converges}$$

2. $\sum_{n=0}^{\infty} \frac{(2n)!}{5n+1}$

$$L = \lim_{n\to\infty} \left|\frac{2(n+1)!}{5(n+1)+1} \cdot \frac{5n+1}{(2n)!}\right| = \lim_{n\to\infty} \left|\frac{(2n+2)!}{5n+6} \cdot \frac{5n+1}{(2n)!}\right|$$

$$= \lim_{n\to\infty} \left|\frac{(2n+2)(2n+1)(5n+1)}{5n+6}\right| = \infty \to L = \infty > 1$$

diverges

3. $\sum_{n=3}^{\infty} \frac{e^{4n}}{(n-2)!}$

$$L = \lim_{n\to\infty} \left|\frac{e^{4(n+1)}}{((n+1)-2)!} \cdot \frac{(n-2)!}{e^{4n}}\right| = \lim_{n\to\infty} \left|\frac{e^{4n+4}}{(n-1)!} \cdot \frac{(n-2)!}{e^{4n}}\right|$$

$$= \lim_{n\to\infty} \left|\frac{e^4}{n-1}\right| = 0 \to L = 0 < 1 \therefore \text{converges}$$

4. $\sum_{n=2}^{\infty} \frac{(-2)^{1+3n}(n+1)}{n^2 5^{1+n}}$

$$L = \lim_{n\to\infty} \left|\frac{(-2)^{1+3(n+1)}((n+1)+1)}{(n+1)^2 5^{1+n+1}} \cdot \frac{n^2 5^{1+n}}{(-2)^{1+3n}(n+1)}\right| = \lim_{n\to\infty} \left|\frac{(-2)^3(n+2)n^2}{(n+1)^2(5)(n+1)}\right|$$

$$= \lim_{n\to\infty} \left|\frac{-8n^2(n+2)}{5(n+1)^3}\right| = 8/5 \to L = 8/5 > 1$$

$\therefore$ diverges

Root Test - converges or diverges?

1. $\sum_{n=1}^{\infty}(\frac{3n+1}{4-2n})^{2n}$

$$L = \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n+1}{4-2n}\right)^{2n}} = \lim_{n\to\infty} \left|\left(\frac{3n+1}{4-2n}\right)^2\right| = \lim_{n\to\infty} \left(\frac{3}{-2}\right)^2 = \frac{9}{4}$$

$L = 9/4 > 1 \therefore$ diverges

2. $\sum_{n=0}^{\infty} \frac{n^{1-3n}}{4^{2n}}$

$$L = \lim_{n\to\infty} \left|\sqrt[n]{\frac{n^{1-3n}}{4^{2n}}}\right| = \lim_{n\to\infty} \left|\frac{n^{1/n}n^{-3}}{4^2}\right| = \left|\frac{(1)(0)}{16}\right| = 0$$

$L = 0 < 1 \therefore$ conv

3. $\sum_{n=4}^{\infty} \frac{(-5)^{1+2n}}{2^{5n-3}}$

$$L = \lim_{n\to\infty} \left|\sqrt[n]{\frac{(-5)^{1+2n}}{2^{5n-3}}}\right| = \lim_{n\to\infty} \left|\frac{(-5)^{1/n+2}}{2^{5-3/n}}\right|$$

$$= \left|\frac{(-5)^2}{2^{5-3/n}}\right| = \frac{25}{32}$$

$L = 25/32 < 1 \therefore$ conv