KEY

11.4 - Comparison Test

Calculus II

Determine if the series converges or diverges

1. $\sum_{n=4}^{\infty} \frac{n^2}{n^3-3}$

$a_n = n^2/n^3 - 3$

$b_n = \frac{n^2}{n^3} = 1/n \rightarrow \sum_{n=4}^{\infty} \frac{1}{n}$

$\rightarrow$ harmonic

$\therefore$ diverges via

DCT $\rightarrow$ direct comparison test

2. $\sum_{n=2}^{\infty} \frac{7}{n(n+1)}$

$a_n = 7/n(n+1)$

$b_n = 7/n^2 \rightarrow \sum_{n=2}^{\infty} \frac{7}{n^2}$ * pseries

$p = 2 > 1$

$\rightarrow$ converges via Comparison Test

$n < n+1$

$\rightarrow n(n) < n(n+1)$

$\frac{7}{n(n)} > \frac{7}{n(n+1)}$

3. $\sum_{n=2}^{\infty} \frac{n-1}{\sqrt{n^6+1}}$ } a_n

$a_n =$

$b_n = \frac{n}{\sqrt{n^6}} = \frac{1}{n^2} \rightarrow \sum_{n=2}^{\infty} \frac{1}{n^2}$

$\rightarrow$ conv, p-series

$p = 2 > 1$

$n > n-1$

$\frac{n-1}{\sqrt{n^6+1}} < \frac{n}{\sqrt{n^6+1}} < \frac{n}{\sqrt{n^6}} = \frac{1}{n^2}$

4. $\sum_{n=3}^{\infty} \frac{e^{-n}}{n^2+2n}$ } a_n

$a_n =$

$b_n =$

$e^{-n} < e^{-3} < 1$

$\frac{e^{-n}}{n^2+2n} < \frac{1}{n^2+2n} < \frac{1}{n^2}$

$\sum_{n=3}^{\infty} \frac{1}{n^2} \rightarrow$ p-series

conv. by comp test

5. $\sum_{n=1}^{\infty} \frac{4n^2-n}{n^3+9}$ } a_n

$a_n =$

$b_n = \frac{4n^2}{n^3} = \frac{4}{n}$

$\sum_{n=1}^{\infty} \frac{4}{n} \rightarrow$ harmonic diverges

$C = \lim_{n\to\infty} \frac{a_n}{b_n}$

$C = \lim_{n\to\infty} \frac{\frac{4n^2-n}{n^3+9}}{\frac{4}{n}} = \frac{4n^3-n^2}{4n^3+36} = 1$

$0 < C = 1 < \infty$

$\rightarrow$ diverges

11.5 - Alternating Series Test

Calculus II

Determine if the series converges or diverges

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{7+2n}$

$b_n = \frac{1}{7+2n}$

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{7+2n} = 0$ ✓

✓ $\frac{1}{7+2n} > \frac{1}{7+2(n+1)}$ → converges

→ plug n+1 in for n

by alt series test

2. $\sum_{n=0}^{\infty} \frac{(-1)^{n+3}}{n^3+4n+1}$

$b_n = \frac{1}{n^3+4n+1}$

$\lim_{n\to\infty} \frac{1}{n^3+4n+1} = 0$

$\frac{1}{n^3+4n+1} > \frac{1}{(n+1)^3+4(n+1)+1}$ → convergent by alt series

3. $\sum_{n=0}^{\infty} \frac{1}{(-1)^n(2^n+3^n)} = \frac{1}{(-1)^n} \cdot \frac{1}{(2^n+3^n)}(-1)^n = \frac{(-1)^n}{(-1)^n(2^n+3^n)} = \frac{(-1)^n}{(2^n+3^n)}$

$b_n = \frac{1}{(2^n+3^n)}$

$\lim_{n\to\infty} \frac{1}{2^n+3^n} = 0$ $\quad \frac{1}{2^n+3^n} > \frac{1}{2^{n+1}+3^{n+1}}$ → convergent by alt series

4. $\sum_{n=0}^{\infty} \frac{(-1)^{n+6}n}{n^2+9}$

$b_n = \frac{n}{n^2+9}$

$\lim_{n\to\infty} \frac{n}{n^2+9} = 0$ ✓

$\frac{n}{n^2+9} > \frac{(n+1)}{(n+1)^2+9}$ → $\frac{n}{n^2+9} > \frac{1}{n+10}$ ✓

→ convergent; by alt series

5. $\sum_{n=4}^{\infty} \frac{(-1)^{n+2}(1-n)}{3n-n^2}$

$b_n = \frac{1-n}{3n-n^2}$

$f(x) = \frac{1-x}{3x-x^2}$

$\lim_{n\to\infty} \frac{1-n}{3n-n^2} = 0$

$f'(x) = \frac{-x^2+2x-3}{(3x-x^2)^2}$ → convergent by alt series