Key

Exam 1 Test Prep - Board Work

1. Find the area of the region enclosed by $y = 3x$ and $y = x^2$

$A = \int_0^3 (3x - x^2)\,dx$

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{9}{2} - 0 = \frac{9}{2}$

*intersection
$x = 0, x = 3$

2. Evaluate $\int_0^{\frac{\pi}{2}} |sin(x) - cos(2x)|dx$

$\sin(x) = \cos(2x)$

$\cos(2x) = 1 - 2\sin^2(x)$

$2\sin^2(x) + \sin(x) + 1 = 0$

$(2\sin^2(x) - 1)(\sin(x) + 1)$

$\sin(x) = 1/2$ or -1

$\pi/6$

$\int_0^{\pi/6} |\sin(x) - \cos(2x)|\,dx + \int_{\pi/6}^{\pi/2} |\sin(x) - \cos(2x)|$

$\int_0^{\pi/6} (\cos(2x) - \sin(x))$

$\left[\frac{1}{2}\sin(2x) + \cos(x)\right]_0^{\pi/6} = \frac{3\sqrt{3}}{2} - 1$

3. Find the volume obtained by rotating the region bounded by x=0, $x = (\frac{1}{3})y^2$ and $0 \le y \le 6$ about the y-axis.

$V = \pi \int_a^b [R(y)]^2\,dy$

$V = \pi \int_0^6 \left(\frac{1}{3}y^2\right)^2 dy$

$= \pi \int_0^6 \left(\frac{1}{9}y^4\right) dy \rightarrow \frac{\pi}{9}\left[\frac{y^5}{5}\right]_0^6 = \frac{864\pi}{5}$

4. The region on $0 \le x \le \frac{\pi}{4}$ bounded above y = 6cos(x) & below y= 6sin(x). Find the volume when this region is revolved about y = -2.

$V = \pi \int_0^{\pi/4} [(6\cos(x) + 2)^2 - (6\sin(x) + 2)^2]\,dx$

$= \pi \int_0^{\pi/4} (36\cos^2(x) - 24\cos(x) + 4) - (36\sin^2(x) + 24\sin(x) + 4)$

$= \pi[18\sin(2x) + 24\sin(x) + 24\cos(x)]_0^{\pi/4} = 6\pi(4\sqrt{2} - 1)$

5. Use cylindrical shells to find the volume by rotating the region bounded by $y = 8x - x^2$ and $y = x^2$ $(0 \le x \le 4)$ about the y-axis.

$r(x) = x$

$8x - x^2 = x^2$

$8x = 2x^2$

$2x^2 - 8x = 0$

$2x(x - 4) = 0$

$V = 2\pi \int_0^4 (x)(8x - 2x^2)\,dx$

$= 2\pi \int_0^4 (8x^2 - 2x^3)\,dx$

$= 2\pi \left[\frac{8x^3}{3} - \frac{x^4}{2}\right]_0^4 = \frac{256}{3}\pi$

6. Suppose it takes 4 J of work to stretch a spring from it's natural length of 40 cm to 50 cm. How much work is required to stretch the spring from 42 cm to 47 cm?

$W = \int_a^b Kx\,dx$

$= \int_0^{.10} \frac{1}{2}Kx^2$

$14 = \frac{1}{2}K(.10)^2$

$K = 800\ N/m$

initial: $.42 - .40 = .02$

final: $.47 - .40 = .07$

$\int_{.02}^{.07} 800x\,dx$

$= 400x^2 \Big|_{.02}^{.07}$

$= 1.80\ J$

Natural length

.40m to .50m

$x = .10\ m$

7. A 400 ft vertical mine shaft contains a cable that weighs 3 lb/ft and is used to lift a 500 lb load of ore. Find the total work required to lift the ore and the entire cable to the top.

$W_{(ore)} = 500(400) = 20{,}000\ ft\text{-}lb$

$W_{(cable)} = \int_0^{400} 3x\,dx \rightarrow \frac{3x^2}{2}\Big|_0^{400} = \frac{3(400)^2}{2} - \frac{3(0)^2}{2} = 240000\ ft\,lb$

$W_T = 20000 + 240000 = 440{,}000\ ft\cdot lb$

8. Find the average value of $h(x) = 6\cos^4(x) \cdot \sin(x)$ on [0, pi]

$\pi - 0 = \pi$

$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{\pi}\int_0^{\pi} 6\cos^4(x)\sin(x)\,dx$

$u = \cos(x)$

$du = -\sin(x)dx$

$\sin(x)dx = -du$

when $x = 0$, $u = 1$

when $x = \pi$, $u = -1$

$= -\int_1^{-1} 6u^4(-du) = -6\int_1^{-1} u^4\,du$

$= 6\int_{-1}^{1} u^4\,du$

$= 6\left[\frac{u^5}{5}\right]_{-1}^{1} = \frac{12}{5}$

$f_{avg} = \frac{1}{\pi}\cdot\frac{12}{5} = \frac{12}{5\pi}$