KEY

11.3 - Integral Test

Calculus II

Determine if the series converges or diverges

1. $\sum_{n=0}^{\infty} \frac{2}{3+5n}$ $\rightarrow \int_0^{\infty} \frac{2}{3+5x}\,dx \rightarrow \lim_{t\to\infty} \int_0^t \frac{2}{3+5x}\,dx$

$\rightarrow \lim_{t\to\infty} \left(\frac{2}{5}\ln|3+5x|\right)\Big|_0^t$

$\rightarrow \left(\frac{2}{5}\ln|3+5t|\right) - \left(\frac{2}{5}\ln|3+5(0)|\right)$

$= \frac{2}{5}\ln|3+5t) - \frac{2}{5}\ln|3| = \infty$

$\rightarrow$ diverges

2. $\sum_{n=2}^{\infty} \frac{1}{(2n+7)^3}$ $\rightarrow \int_2^{\infty} \frac{1}{(2x+7)^3}\,dx \rightarrow \lim_{t\to\infty} \int_2^t \frac{1}{(2x+7)^3}\,dx$

$\rightarrow \lim_{t\to\infty} \left(-\frac{1}{4}\cdot\frac{1}{(2x+7)^2}\right)\Big|_2^t$

$\rightarrow \lim_{t\to\infty} \left(-\frac{1}{4}\cdot\frac{1}{(2t+7)^2} + \frac{1}{4}\cdot\frac{1}{(2(2)+7)^2}\right) = 1/484$ $\rightarrow$ convergent

3. $\sum_{n=1}^{\infty} ne^{-n^2}$ $\rightarrow \int_1^{\infty} xe^{-x^2}\,dx \rightarrow \lim_{t\to\infty} \left[-\frac{1}{2}e^{-x^2}\right]_1^t$

Let:
$u = -x^2$
$du = -2x\,dx$
$x\,dx = -\frac{1}{2}du$

$\rightarrow \lim_{t\to\infty} \left(-\frac{1}{2}e^{-t^2} - \left(-\frac{1}{2}e^{-1^2}\right)\right) = \lim_{t\to\infty}\left(-\frac{1}{2e^{t^2}} + \frac{1}{2e}\right)$

$= 0 + \frac{1}{2e} = 1/2e$

$\rightarrow$ convergent

4. $\sum_{n=1}^{\infty} \frac{1}{3\sqrt{n^5}} \to \sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$ *p-series : $p = \frac{5}{3}$

$p > 1 \to$ converges

$\frac{5}{3} > 1$ ∴, series converges

5. $\sum_{n=2}^{\infty} \frac{1}{n \ln x} \to \int_2^t \frac{1}{x \ln(x)} dx \to \lim_{t \to \infty} \int_2^t \frac{1}{x \ln(x)} dx$

Let:

$u = \ln(x)$

$du = \frac{1}{x} dx$

$\to \lim_{t \to \infty} \int_{\ln(2)}^{\ln(t)} \frac{1}{u} du$

$\to \lim_{t \to \infty} \left[\ln|u| \right]_{\ln(2)}^{\ln(t)}$

$\to \lim_{t \to \infty} \left(\ln|\ln(t)| - \ln|\ln(2)| \right)$

$\to \infty - \ln(\ln(2)) = \infty$

$\to$ diverges